

*PACKING VOLUME CONCEPT  
FOR  
AGGREGATES*

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*Joint  
Highway  
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*by  
E. Tons  
W. H. Goetz*

*PURDUE UNIVERSITY  
LAFAYETTE INDIANA*



Technical Paper

PACKING VOLUME CONCEPT FOR AGGREGATES

To: G. A. Leonards, Director  
Joint Highway Research Project

September 27, 1967

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From: H. L. Michael, Associate Director  
Joint Highway Research Project

Project No.: C-36-6K

Attached is a paper entitled "Packing Volume Concept for Aggregates" by Egons Toms and W. H. Goetz of our staff, which has been prepared for presentation to the Annual Meeting of the Highway Research Board in Washington, D. C., January 1968. The data were obtained by Mr. Toms in connection with the HPR-1 (5) research study on "Flow in Aggregate-Binder Mixes" and have been presented to the Board previously in the form of a progress report.

The main conclusion of the paper is that grading aggregates according to packing volume distribution rather than sieve size distribution should give a unifying approach to mix design using diverse types of aggregates (rounded gravel, crushed rock, etc.).

The paper is presented to the Board for approval of the presentation as indicated. The paper will also be forwarded to the ISEC and the HPR for their review, comments and approval of such publication.

Respectfully submitted,

*Harold L. Michael/jgm*  
Harold L. Michael  
Associate Director

HLM:mf

Attachment

Copy: F. L. Ashbaugh  
W. L. Dolch  
W. H. Goetz  
W. L. Grecco  
G. K. Hallock  
M. E. Harr

R. H. Harrell  
J. A. Havers  
V. E. Harvey  
J. F. McLaughlin  
F. B. Mendenhall  
R. D. Miles  
J. C. Oppenlander

C. F. Scholer  
M. B. Scott  
W. T. Spencer  
H. R. J. Walsh  
K. B. Woods  
E. J. Yoder



Technical Paper

PARKING VOLUME CONCEPT FOR AGGREGATION

by

Egon Tonn, Research Assistant

and

H. D. Goets, Professor of Highway Engineering

Joint Highway Research Project

Project: C-35-GM

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## Technical Report

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## PACKING VOLUME CONCEPT FOR AGGREGATES

### INTRODUCTION

Although aggregates found in particle composites such as bituminous concrete, portland cement concrete and untreated granular masses are sometimes described by summation of volumes and/or surface areas, usually gradation by sieve size is employed as a primary guide in combining and designing mixes for different service conditions. It has been recognized that equal grading by sieves and batching by weights do not assure similar bulk properties when dealing with diverse types of naturally occurring (gravel) and artificially produced (crushed rock, etc.) aggregates. Yet, the difficulty in describing particles by measurable parameters of a general nature has prevailed because definitions of descriptive characteristics such as "volume," "surface area," or "surface roughness" are not as yet agreed upon for an irregular particle.

The primary purpose of this work was to search for the least number of measurable parameters for individual particles (aggregates) which would be predictive to the bulk behavior of particle composites with and without a binder (such as bitumen, clay, etc.).

The work reported in this paper involves selected rocks without a binder, approximately  $3/4$ " to  $1/8$ " in "size." The main emphasis in the laboratory work was placed on "one-size" particles, which may be represented in certain subbase, base and binder courses in highways.



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## LITERATURE REVIEW

A number of studies to characterize pieces of rock have been made. The main factors of apparent importance which have emerged are: a) particle geometry (sometimes called shape or sphericity), b) angularity (roundness) and c) surface roughness (texture). There are two recent and informative summaries by Grenhaug (1)<sup>1</sup> and Mather (2) based on about two hundred references discussing the various parameters. A limited number of these references are included in the Bibliography. In the work reported here, the main emphasis was placed on a quantitative approach to describe particle geometry, volume, surface roughness and particle behavior in bulk.

### Particle Geometry

Apparently there are four factors affecting the shape of an aggregate particle at the time when it is used: a) type of rock; b) geologic history; c) type of crushing, and d) sizing operation (1) (3) (4) (5) (6).

A number of qualitative terms are used to picture a piece of rock (rounded, irregular, flaky, rods, discs, blades, equidimensional). There have been attempts to quantify particle dimensions by a so-called Sphericity Factor S: (7) (8)

$$S = \sqrt[3]{\frac{\pi/6 \, d^3}{\pi/6 \, \ell^3}} = \frac{d}{\ell} \quad (1)$$

Here d is taken as the diameter of a sphere of the same volume as the rock piece and  $\ell$  is the long dimension of the particle. Further improvement in defining the geometric shape of a particle is achieved by using three descriptive measurements; namely, long ( $\ell$ ), short (s) and medium (m) dimensions (9) (10) (11). For instance, in ASTM Designation C-125 (11)

<sup>1</sup> Numbers in parentheses refer to references in the Bibliography.



the ratios of  $\ell/m$  ( $\ell/w$ ) and  $m/s$  ( $w/t$ ) are adapted to classify rock pieces into four categories ranging from "flat" to "elongated." The values of the ratios for differentiating between the various classes are set arbitrarily.

The measurement of three "diameters" of a particle suggests the geometric form of an ellipsoid. This idea offers a great deal of flexibility. The possibility of using an ellipsoid has apparently not been much explored. Mackay (12), in connection with his work on radii of curvature measurements, uses the concept of a perfect ellipsoid and the degree of departure from this shape.

In research with particles in bulk, the effect of aggregate shape has been investigated by a number of researchers. Since the shape factor is hard to separate from other factors such as angularity, surface roughness and material properties, it is difficult to judge the true influence of particle geometry on mass density and other properties. This may account also for the apparent contradictions summarized by Gronhaug and Hether (1) (2).

#### Angularity or Roundness

Angularity or roundness is often described in qualitative terms such as angular, subangular, subrounded, rounded or well-rounded (13). Another more quantitative way of describing roundness is to take the ratio of the average radius of curvature ( $r$ ) of the corners ( $n$ ) to the radius of the maximum inscribed circle ( $R$ ) of the rock piece (14) (15) (16):

$$\text{Roundness} = \frac{1}{n} \sum \frac{r}{R} \quad (\text{XI})$$

Since in the case of crushed angular pieces the radii of curvature are very small and difficult to measure accurately, actual angles of the

The results of the study are presented in the following table. The data are based on the results of the study of the "National" and "International" groups. The results are presented in the following table.

sharp edges can be determined in addition to the radii of the rounded-off corners (12).

In addition to the above direct methods of measurement, experimental determinations have been made using masses of particles for relative comparisons. These include refined sieving through calibrated openings, measurement of voids in bulk, measurement of angle of repose and others.<sup>2</sup> The results are varied and angularity appears to be almost always confounded with change in particle shape and roughness. Furthermore, the measurement of angularity as such cannot be used directly to calculate or predict any effects on the behavior of particles in bulk. These effects have to be determined experimentally.

#### Surface Texture

The existence of surface texture or roughness of aggregate surfaces is easy to grasp but hard to measure. One way to express roughness quantitatively is by using mean surface and deviation from it (17). There are several publications on surface texture and finish, including devices and methods for measurement (13) (17) (18) (19) (20). There appears to be no definite agreement on classification of roughness except in qualitative terms (rough, smooth, furrowed, grooved, scratched, ridged, pitted, dented, striated, frosted, etched, etc.). Measurement of actual surface area of a certain polished and rough limestone has shown that the rough area was about three times greater than the polished area (21). Blanks (22) has pointed out that there are two kinds of surface roughness: abrupt and undulatory.

<sup>2</sup> The reader is again referred to references (1) and (2) for a more detailed discussion of this subject.



A simple quantitative method for measuring surface roughness for smooth, level surfaces has been proposed by Bikerman (26). He coated flat sawed rock plates with asphalt, scraped the excess down to the stone and used the amount of asphalt left as an indicator of surface roughness (and absorption).

#### Angularity and Roughness Combined

At least intuitively, the shape or geometry of an aggregate piece is a separate parameter. There is a question, however, whether angularity and roughness do not overlap, especially in the case of crushed rock. Gronhaug (1) proposes to combine angularity and texture (roughness) into one term: form. Another, possibly unifying, term would be rugosity (17). Here the adjective "rugged" which stands for rough, uneven, jagged, ridged, or wrinkled, seems to be applicable to irregular particles of aggregates. (It may also encompass some of the surface voids of the rock).

#### Comparison of Irregular Particles with Spheres in Bulk

The use of sphericity and roundness suggests a general trend to equate or compare irregular particles with spheres. Porosities ( $n$ ) or voids in a mass of perfect spheres, packed in a certain order are well known (24). Three cases for one-size spheres are given here:

(a) simple cubical packing

$$n = 1 - \frac{\pi}{6} d^3 \left( \frac{1}{d} \times \frac{1}{d} \times \frac{1}{d} \right) = 1 - \frac{\pi}{6} = 0.476 \text{ (47.6\%)}$$

(b) cubic tetrahedral packing

$$n = 1 - \frac{\pi}{6} d^3 \left( \frac{2}{d\sqrt{3}} \times \frac{1}{d} \times \frac{1}{d} \right) = 1 - .605 = .395 \text{ (39.5\%)}$$





(c) densest packing - tetrahedral

$$n = 1 - \frac{\pi}{6} d^3 \left( \frac{2}{d\sqrt{2}} \times \frac{1}{d} \times \frac{1}{d} \right) = 1 - .740 = .260 \text{ (26.0\%)}$$

For randomly packed spheres and irregular particles, the porosities usually vary between the loose and the dense case.

#### Sliding Friction Between Particles

During packing of rocks, either by gravity-flow or by some mode of densification such as vibration, some relative movement between particles may take place. The actual contact area between two particles (rough or smooth) is small and in the order of about 0.01 percent of the apparent contact area (23). For two hard rock pieces:

$$F = W \frac{s}{y} \quad (\text{III})$$

where  $F$  = force to drag one particle along the surface of another

$W$  = load on the particle (contact)

$s$  = shear resistance

$y$  = yield value

The ratio  $s/y$  should be nearly independent of the nature of the rock itself, since  $s$  and  $y$  tend to vary together.<sup>3</sup> Thus <sup>for</sup> rocks with clean surfaces, the force  $F$  should be dependent on the load only. In other words, a mass of one-size particles subjected to identical load  $W$  (compaction) <sub>s</sub> should respond in/similar manner.

#### Free Fall and Vibratory Compaction

From physics, the velocity in free fall neglecting air friction is (27):

$$V = \sqrt{2gx} \quad (\text{IV})$$

<sup>3</sup> Adamson (23), page 332 argues this point for metals.



where  $V$  = velocity of particle  
 $g$  = gravitational constant  
 $x$  = distance of fall

In other words, if two masses of particles are "poured" from identical heights into a container, their velocities will be about the same regardless of the individual particle mass.

In vibratory compaction (sinusoidal) D'Appolonia (25) claims that the peak acceleration in  $g$ 's is important and, in order to get noticeable compaction, acceleration over one  $g$  is necessary. The useful equation for relating frequency and amplitude to acceleration is:

$$f^2 = \frac{a_g}{.102A} \quad (v)^4$$

where  $f$  = frequency, cps

$a_g$  = acceleration in  $g$ 's

$A$  = peak amplitude

#### Summary of Literature

In this brief survey only selected groups of references were recalled. These are concerned primarily with concepts relative to measurement of particle properties and behavior. No attempt has been made to go into a discussion of published data based on experimental evidence involving several factors at the same time.

The survey of the literature has made it apparent that:

(a) There is no unified agreement on the parameters of importance for quantitative characterization of rock particles.

(b) There is a need to find and tie in such characteristics to the behavior of particles with different compositions and sizes in bulk (bulk densities, flow characteristics, etc.).

\* Since equation V in Reference (25) has a minor error and no derivation is given, it is derived in Appendix I of this paper.



## THEORETICAL CONSIDERATIONS

This section describes the background reasoning leading to laboratory measurements and particle packing volume as a proposed useful concept when dealing with an aggregate mass. The main emphasis is placed on monovolume (one-size) particles, about  $\frac{1}{4}$ , 0.4 and 0.04 cc in volume ( $\frac{3}{4}$ ,  $\frac{3}{8}$  and  $\frac{1}{8}$  inch "size").

### Particle Volume - Ellipsoid Geometry

The bulk volume of a number of particles in a container is, among other things, a function of the volumes of each of them. To start with, it is assumed that the volume which a particle occupies in a mass of other particles largely determines the density and the voids in bulk and that therefore this volume is important as far as the response of the composite to various forces is concerned. The problem at hand is to attempt to define the volume of a particle, especially if it is irregular in shape as well as rough (high rugosity).

In order to define the volume of any particle, it is convenient to have a geometric form which lends itself to numerical description and analytical manipulation. As pointed out in the literature survey, the measurement of long, medium and short dimensions of a particle is not a new idea. Since in the field of aggregates there are practically no cubes, spheres, rods or other regular shapes, why not try to fit an ellipsoid for all types of particles as a geometric form?

The volume of an ellipsoid is simply:

$$V = \frac{\pi}{6} \ell_{ms} = 0.524 \ell_{ms} \quad (VI)$$

### Packing Volume of a Particle

All surfaces of particles possess some kind of roughness. The peaks or asperities of the roughness are spaced randomly. For example,



if two pieces of crushed limestone are in contact with each other, the peaks and the valleys will not be able to mesh like two carefully cut gears. Instead, the particles will touch one another at the high spots, and only a small portion of the stress will be in contact (23). Therefore, the volume which a piece of rock occupies in a mass of other particles encompasses not only the volume of solids and internal voids, but also the volume of the dips and valleys of the particle surface which may be called "outside voids" (see Figure 1a). These outside voids are primarily a function of the rugosity of a surface. The term "packing volume" when applied to a particle is used in this study as that volume which the particle occupies in a mass of particles or:

$$V_p = V_s + V_i + V_o \quad (\text{VII})$$

where  $V_p$  = packing volume

$V_s$  = volume of solids of the particle

$V_i$  = volume of internal voids

$V_o$  = volume of outside voids or surface irregularities

The packing volume can be pictured as a volume enclosed by a dimensionless, flexible membrane stretched along the surface of a rock, touching the asperities (Figure 1a).

In the laboratory, packing volume can be measured by immersing the rock in asphalt, removing the excess asphalt down to the peaks of the surface and weighing the piece in air and water (see Figure 1b):

$$V_p = \frac{W_t - W_w}{G_w} \quad (\text{VIII})$$

where  $W_t$  = total weight, rock plus asphalt, in air

$W_w$  = weight in water

$G_w$  = unit weight of water





The weight of a rock piece to give a certain desired packing volume for practical application can be derived as follows (see Figure 1b):

$$V_p = V_s + V_v + V_a \quad (\text{IX})$$

$$V_p = \frac{W}{G_{s+v}} + V_a \quad (\text{X})$$

$$W = G_{s+v} (V_p - V_a) \quad (\text{XI})$$

where  $V_v$  = internal and surface voids unfilled with asphalt

$V_a$  = volume of asphalt after scraping

$W$  = weight of dry rock piece

$G_{s+v}$  = specific gravity of solids plus voids including those under the asphalt coating

$V_p$  = packing volume

$V_s$  = volume of solids of the particle

The volume of asphalt,  $V_a$ , will depend on the surface area  $A$  and surface roughness  $R$  of the rock piece.<sup>5</sup> Therefore, Equation XI can be re-written:

$$W = G_{s+v} (V_p - AR) \quad (\text{XII})$$

This simply says that for a given packing volume  $V_p$ , the weight of the rock is a function of specific gravity  $G_{s+v}$ , surface area  $A$  and surface roughness or rugosity  $R$ .

The equation for  $G_{s+v}$  is:

$$G_{s+v} = \frac{W}{\frac{W_t - W_v}{G_w} - \frac{W_a}{G_a}} \quad (\text{XIII})$$

$$^5 R = \frac{\text{Volume asphalt on rock after scraping, cc}}{\text{Surface area of rock, cm}^2} = \frac{V_a}{A} \quad (\text{cm})$$



where  $W$  = weight of the rock piece (or pieces)

$W_t$  = weight of the rock piece + asphalt in air

$W_w$  = weight of the rock piece + asphalt in water

$W_a$  = weight of asphalt

$G_a$  = specific gravity of asphalt

$G_w$  = unit weight of water

The value  $G_{s+V}$  is constant for a given aggregate piece provided a certain procedure is followed just as in any test for specific gravity of aggregates. However, if two laboratories use two different methods and obtain two different  $G_{s+V}$  values, equation XII still holds, since rugosity  $R$  changes in unison with  $G_{s+V}$  (see Figure 1b). The knowledge of  $G_{s+V}$  may be useful for obtaining rugosity  $R$  factors without resorting to scraping.

As mentioned in the literature review, for a given aggregate the surface rugosity  $R$  is higher for larger pieces as compared to smaller ones. During crushing operations cracks propagate along the path of least resistance, leaving fine surface roughness superimposed on longer undulating roughness.<sup>6</sup> The smaller the rock, the less the inclusion of larger undulations are included and the smaller should be the rugosity factor  $R$ .

#### Grading by Sieves and Packing Volume

Grading by sieves alone does not assure good control of particle packing volume. The volume of an ellipsoid is  $V = \pi/6 \cdot d_{ms}$  and the medium and short dimensions are primarily responsible for passage through a square-hole sieve. The relationship between sieve opening (size)  $R$  and the magnitudes of  $m$  and  $s$  to pass a square hole is:

<sup>6</sup> In reality there is probably a continuous distribution of fine roughness going into undulations.

where  $W$  = weight of the rock piece (in grams)

$W_a$  = weight of the rock piece + capsule in air

$W_w$  = weight of the rock piece + capsule in water

$V$  = volume of rock piece

$\rho$  = density of rock piece

$\rho_w$  = density of water

The volume of the rock piece is determined by the difference in weight of the rock piece + capsule in air and in water, divided by the difference in density of air and water.

The density of the rock piece is determined by the weight of the rock piece divided by the volume of the rock piece.

The weight of the rock piece is determined by the weight of the rock piece + capsule in air, minus the weight of the capsule in air.

The weight of the capsule in air is determined by the weight of the capsule + water in air, minus the weight of the water in air.

The weight of the capsule + water in air is determined by the weight of the capsule + water in water, plus the weight of the water in air.

The weight of the water in air is determined by the weight of the water in water, minus the weight of the water in air.

The weight of the water in water is determined by the weight of the water in air, plus the weight of the water in water.

The weight of the water in air is determined by the weight of the water in water, minus the weight of the water in air.

The weight of the water in water is determined by the weight of the water in air, plus the weight of the water in water.

The weight of the water in air is determined by the weight of the water in water, minus the weight of the water in air.

The weight of the water in water is determined by the weight of the water in air, plus the weight of the water in water.

$$H = \sqrt{m^2 + s^2} \quad (XIV)$$

The volume of such a particle (ellipsoid) would be

$$V = \frac{\pi}{6} \ell m s$$

$$m = \frac{6V}{\pi \ell s}$$

from equation XIV

$$m = \sqrt{H^2 - s^2}$$

or

$$\sqrt{H^2 - s^2} = \frac{6V}{\pi \ell s}$$

or

$$\frac{V}{\ell} = \frac{\pi}{6} s \sqrt{H^2 - s^2}$$

where always  $\ell > s$ .

For a given size  $H$  and value  $S$

$$\frac{V}{\ell} = K \quad \text{or} \quad V = K \ell$$

This shows quantitatively that aggregates passing a given sieve (and having identical  $m$  and  $s$  values) will have uncontrolled volumes directly proportional to the length  $\ell$  of the rock piece (see Figure 2). Thus the particle volume distributions should be different for different rocks of identical sieve size. However, for a given aggregate, it should be possible to use sieve grading to predict volume grading through correlation factors.

#### Packing Densities of Perfect Ellipsoids

Packing densities and voids (porosity) for perfect spheres under certain configurations have often been used for comparisons in particle studies. The volume of a sphere is  $V_s = \frac{\pi}{6} d^3$ , of an ellipsoid,  $V = \frac{\pi}{6} \ell m s$ .



For the case of one-size smooth spheres in simple cubical packing, the porosity  $n = 47.6$  percent. If, instead, ellipsoids of identical  $\ell$ ,  $m$ , and  $s$  values are packed in a similar manner:

$$n = 1 - \frac{\pi}{6} \ell m s \left( \frac{1}{\ell} \times \frac{1}{m} \times \frac{1}{s} \right) = 0.476 \text{ (47.6\%)}$$

For spheres in cubic-tetrahedral packing  $n = 39.5$  percent, and for ellipsoids in similar arrangement,

$$n = 1 - \frac{\pi}{6} \ell m s \left( \frac{2}{\ell\sqrt{3}} \times \frac{1}{m} \times \frac{1}{s} \right) = .395 \text{ (39.5\%)}$$

Finally, in the densest tetrahedral packing for spheres  $n = 26.0\%$ .

The packing for ellipsoids is similar:

$$n = 1 - \frac{\pi}{6} \ell m s \left( \frac{2}{\ell\sqrt{2}} \times \frac{1}{m} \times \frac{1}{s} \right) = .260 \text{ (26.0\%)}$$

From the above calculations it is apparent that dense, loose, and intermediate packing of perfect equidimensional ellipsoids give voids (porosities) identical to those between packed spheres (for derivation of the above equations see Appendix 2).

So far, consideration has been given to the shape (ellipsoid) and rugosity (surface roughness) of the particle. It should be pointed out that rugosity as measured by scraping may also be influenced by angularity (the more angular the rock, the higher the rugosity). Sharp corners of rock, however, are not accounted for in the packing volume concept.

#### Bulk Volumes

Calculations have shown that volumes of voids for one-diameter spheres and equidimensional ellipsoids are identical under ideal packing conditions. Since the ellipsoid gives a good approximation for the geometry of irregular particles, it is conceivable that the particles will also pack similarly to spherical particles in a composite.





The void content (or porosity) of a mass of small or large one-volume particles should be the same, regardless of the type of rock and shape of particle, just as it is with ideal spheres. Thus the ratio of the number of small particles  $N_1$  to the number of large ones,  $N_2$ , should be indirectly proportional to their packing volumes  $V_{p1}$  and  $V_{p2}$ :

$$\frac{N_1}{N_2} = \frac{V_{p2}}{V_{p1}}$$

(XV)

$$N_1 V_{p1} = N_2 V_{p2}$$

When packing or compacting different kinds of particles, identical procedures are necessary to obtain comparable results. Thus, for example, when a mass of rocks is "poured" into a given mould or container, the rocks must be deposited from a similar height and at similar rates of speed (12). If vibratory compaction is used, the peak acceleration must be identical (25).

Finally, it must be pointed out that the porosity, or voids, as considered here is not the absolute porosity of the bulk since the basis of the "solid" volume is packing volume, which includes surface roughness or surface voids. Thus, one-volume rounded gravel and crushed stone may have identical packing porosities in a mass, but the amount of liquid such as water or mercury to fill the aggregate voids would be greater in the case of the crushed limestone.



## EXPERIMENTAL WORK

The reasoning developed by theoretical considerations was tested in the laboratory using three types of rocks (crushed limestone, crushed gravel, and rounded gravel) with three distinct packing volumes about one decade apart (4 cc, 0.4 cc and 0.04 cc). In terms of "sizes" the rocks were about 3/4, 3/8 and 1/8 inches respectively (see Figure 3). In addition, comparative measurements were made using 1/2-inch smooth glass spheres (marbles). The surface rugosity and geometric shape were measured, packing volumes were calculated and weights for identical bulk volumes were predicted for the various rocks and sizes. Loose bulk volumes and volumes after vibratory compaction were measured and compared to check the validity of the packing volume concept.

### Description of Aggregates

The three aggregates were selected on the basis of differences in rugosity (crushed versus rounded) and composition (sedimentary versus mixed). These three types are also frequently used in highway construction. The crushed gravel and the rounded gravel came from the same source.

It is apparent that one-size or one-volume particles exist only in theory. Even smooth, one-size glass spheres (marbles) do not have identical diameters. It is also impossible to produce one-volume rock particles, and therefore, the three categories of rock volumes are actually mean volumes with a controlled standard deviation and about equal coefficient of deviation.

The 0.04 cc (1/8") rocks were obtained by dividing the fraction between sieves No. 4 and No. 6 into portions retained on sieves No. 5 and No. 6. Then these two fractions were combined accordingly to get similar coefficients of deviation,  $D^7$ , for the three types of rocks, based on

$$D^7 = \frac{\text{Standard Deviation}}{\text{Average Weight}} \times 100$$



weights of particles. A convenient D was at about 15 percent. Similar handling of the 0.4 cc and 4 cc rocks, using appropriate sieves, gave a desired D = 15 percent in each case.

#### Measurement of Rugosity and Packing Volume

As mentioned before and shown by Figure 1b, packing volume of particles can be measured without the numerical determination of rugosity. It is convenient, however, to have available the characteristic relationship between rugosity and different particle sizes (volumes) for a given rock because it provides a basis for calculating particle packing volumes and weights for other "sizes" than those used in the actual determination (see Equations X and XII). Rugosity value is also needed when calculating the amount of a binder, such as asphalt, to be mixed with aggregate. The primary reason for measuring rugosity here is to show that it adds to the particle volume in bulk.

Figure 4 shows the rugosity as it changes with the volume of each rock. To obtain the curves, rock pieces were drawn at random from a mass of other pieces of the same size and then washed, dried, weighed, and heated to 300 F in a compartmentalized container. They were then covered by a 60-70 penetration asphalt at 300 F for thirty minutes after which each coated rock was dipped in ice water. The excess asphalt was scraped off each piece, down to the peaks of roughness.

The scraping was done with a razor blade, applying its straight edge and avoiding use of the corners (26).<sup>8</sup> This operation was tedious and required some patience and skill. After scraping, crushed rock and rounded gravel look very much alike except for some sharp angles of the former.

<sup>8</sup> This method was first used by J. H. Bikerman on smooth stone plates.



The particles were weighed again in air and in water, giving direct measurement of their packing volumes. Furthermore, the three dimensions  $\ell$ ,  $m$  and  $s$  for each rock piece were measured and their "membrane" surface area was calculated using the simplified equation of prolate spheroids:

$$A = \frac{1}{2} \pi d \left( d - \frac{\ell}{K} \sin^{-1} K \right)$$

where

$A$  = surface area of particle ("membrane" area)

$$d = \frac{\ell + s}{2}$$

$$K = \left( \frac{\ell^2 - d^2}{2\ell d} \right)^{1/2}$$

In practice the areas for each rock piece were obtained using a graph identical to Figure 5 but on an expanded scale. Using the above data, rugosity values were calculated for different sizes and kinds of rocks (Figure 4):

$$\text{Rugosity} = \frac{\text{Amount of asphalt on rock, cm}^3}{\text{Surface area of rock, cm}^2}$$

The  $m$ ,  $\ell$  and  $s$  values used in surface area calculations for each rock were also useful for calculating packing volumes and comparing them with those measured by the water displacement method. Statistical difference analysis indicated that direct measurement using the assumed shape of ellipsoid is in good agreement with the results of the volume-by-water-displacement method (for detailed analysis see Appendix 3). This may suggest another method for measuring packing volumes of particles.

In order to determine differences in the shape of ellipsoids, comparisons were also made among  $\ell/s$ ,  $\ell/m$  and  $m/s$  ratios for various fractions of the same rock; they are shown in Figure 6. The curves indicate a slight tendency for the 0.4 cc rocks to have higher ratios





compared to the smaller and larger rocks of the same kind. A numerical analysis in Appendix 4 indicates that the differences do not appear important.

#### Sieve Size and Particle Volume

As pointed out previously, the particle volume distribution of crushed limestone and rounded gravel (or any two aggregates) is expected to be different <sup>even</sup> if taken from the same sieve size fraction. Figure 7 shows an example of packing volume distribution curves obtained for a 1/2" - 3/8" crushed limestone and the same size gravel. In the case of the limestone, there is a tendency for the average volume of the particles to be smaller. It is expected that each type of rock from a given quarry (factor of constants remains constant), sieved following a given procedure will result in a characteristic particle volume distribution on each sieve. Once this distribution is known, aggregates could be combined on the basis of particle packing volume distribution using sieve grading and correlating according to the "packing volume formula."

#### Number of Contact Points

The number of contact points in a simple cubical packing and in a tetrahedral closest packing of spheres is 6 and 12 respectively. The same factor applies to ellipsoids in similar loose and dense packings. The number of contact points for the fine groups of one-value aggregates were determined at one particular mass density, using a plain coating for detection. The procedure involved mixing about 500 cc. of half a quart of rocks at 300 F with just enough 60-70 penetration asphalt to fill the volume of rugosity. The coated rocks were then placed in a container so as to obtain about equal porosities. Then the mass was cooled to 0 F, the rocks separated and the contact points counted. An example of a



distribution curve is shown in Figure 8. Again, no significant differences in number of contacts between the kinds and sizes of rocks were found (see Appendix 5). It is expected that the number of contact points increases with increased compaction of the particles (24).

#### Identical Bulk Volumes with Identical $\sum V_p$ of Rocks

As given by equation II, the packing volume  $V_p$  of each individual rock can be calculated:

$$V_p = \frac{W}{G_{s+v}} + AR$$

The number  $N$  of particles needed to give a certain bulk packing volume  $\sum V_p$  of a mass of particles is:

$$N = \frac{\sum V_p}{V_p}$$

The total weight  $\sum W$  of such a mass of  $N$  particles would be

$$\sum W = NW = \frac{\sum V_p}{V_p} W$$

or

$$\sum W = G_{s+v} \left( V_p + AR \right) \frac{\sum V_p}{V_p} \quad (XVI)$$

Equation XVI permits the calculation of how much by weight of a certain "size" of rock is to be taken to obtain a given packing volume  $\sum V_p$  for a mass of particles.

If Equation XVI holds and if sliding friction is similar with the three rocks tested, they should have similar bulk volumes for identical  $\sum V_p$ . This theory was tested by two methods: free fall and vibratory compaction.



In the free-fall method, identical  $\sum V_p$  of the three rocks and three sizes (nine batches altogether) were prepared to give approximately 85 cubic inches (1400 cc) of loose bulk volume. The dry, clean rocks were then poured into a cylindrical container 5 inches in diameter and 5 inches high from an average height of 3 inches and within a time interval of 10 seconds. The resulting bulk volumes were then determined for each of the rock types and sizes. Graphical comparisons are given in Figure 9. Statistical analysis showed that there was no significant difference in bulk volumes thus obtained.

In another series of tests sinusoidal vibration was applied to the various rocks in bulk with peak acceleration of 1.5 times gravity and without surcharge. The frequency chosen was 20 cps and the bulk-mass volumes of the rocks were measured at 1, 10, 100 and 1000 cycles of full vibration. The results are summarized in Figure 10 and the analysis is given in Appendix 6 for the cases of 1 and 1000 cycles. It was convenient here to compare porosities  $n_p$  instead of bulk volumes.

The measurements again showed that all rocks had similar densification trends. The bulk volumes and porosities  $n$  obtained at each of the indicated cycles were similar for a given sum of packing volumes ( $\sum V_p$ ) regardless of rock type or size; this was expected from the theoretical considerations.

As an additional check, 1/2 inch marbles with  $R = 0$  and the same  $\sum V_p$  were included in both of the above "compaction" tests. Marbles behaved similarly to the various one-size rocks.



## DISCUSSION OF RESULTS

The primary goal of this study was to search for a unifying approach to the characterization of aggregate particles of various kinds and sizes. The scope has been limited to three types and three sizes of aggregates in the coarse aggregate range.

The central hypothesis was that the volume which a particle, large or small, angular or rounded, smooth or rough, occupies in a mass of other particles is an important characterizing factor as far as bulk properties under a defined "compaction" are concerned. The test results show that a particle packing volume concept is useful in defining the characteristic space a piece of rock occupies in a bulk. This packing volume can be obtained using Equation X:

$$V_p = \frac{W}{G_{s+v}} + V_a$$

where  $G_{s+v}$  is defined by Equation XIII.

However, when dealing with bulk density (or porosity) another type of specific gravity for a rock piece based on packing volume  $V_p$  may be useful. This could be called packing specific gravity,  $G_p$ . If  $W$  is the dry weight of a rock piece and  $V_p$  is its packing volume, then

$$G_p = \frac{W}{V_p} \quad (\text{XVII})$$

Numerically,  $G_p$  would be the lowest of all commonly used specific gravity values since the volume includes surface voids. It would be constant for one given volume of rock, but would vary with rock size and type because surface roughness and surface area, which are functions of rock size and type, determine surface voids. From Equation XVI a weight-volume relationship for a number of particles taken together is:





$$\sum W = \sum V_p G_p \quad (\text{XVIII})$$

If a given total packing volume  $\sum V_p$  (say  $\sum V_p = 1000$  cc) of a number of particles of any of the nine individual rock-size groups were desired and designated as:

$$\sum V_{p1}, \sum V_{p2} \dots \dots \sum V_{p9}$$

then the total weight  $\sum W$  needed to give these constant volumes could be calculated using Equation XVIII. It is apparent that generally:

$$\sum W_1 \neq \sum W_2 \neq \dots \neq \sum W_9$$

However, if identical free fall or vibratory "compaction" is employed to densify the above nine batches of one size rock, the bulk volumes  $V_B$  will be equal or

$$V_{B1} = V_{B2} = \dots = V_{B9}$$

This also leads to the conclusion that weight per unit volume of these particles will be different depending on  $G_p$  and the type of compaction. The packing volume concept at this stage of development permits the calculation of "one-size" rock weights which will produce identical bulk volumes under identical compaction.<sup>9</sup>

Thus, laboratory findings support the general line of theoretical considerations discussed previously. Angularity did not prove to be a distinctive feature, even though some of this is taken care of by the

<sup>9</sup> This concept is also expected to permit the prediction of flow of an aggregate mass mixed with a binder and subjected to load.

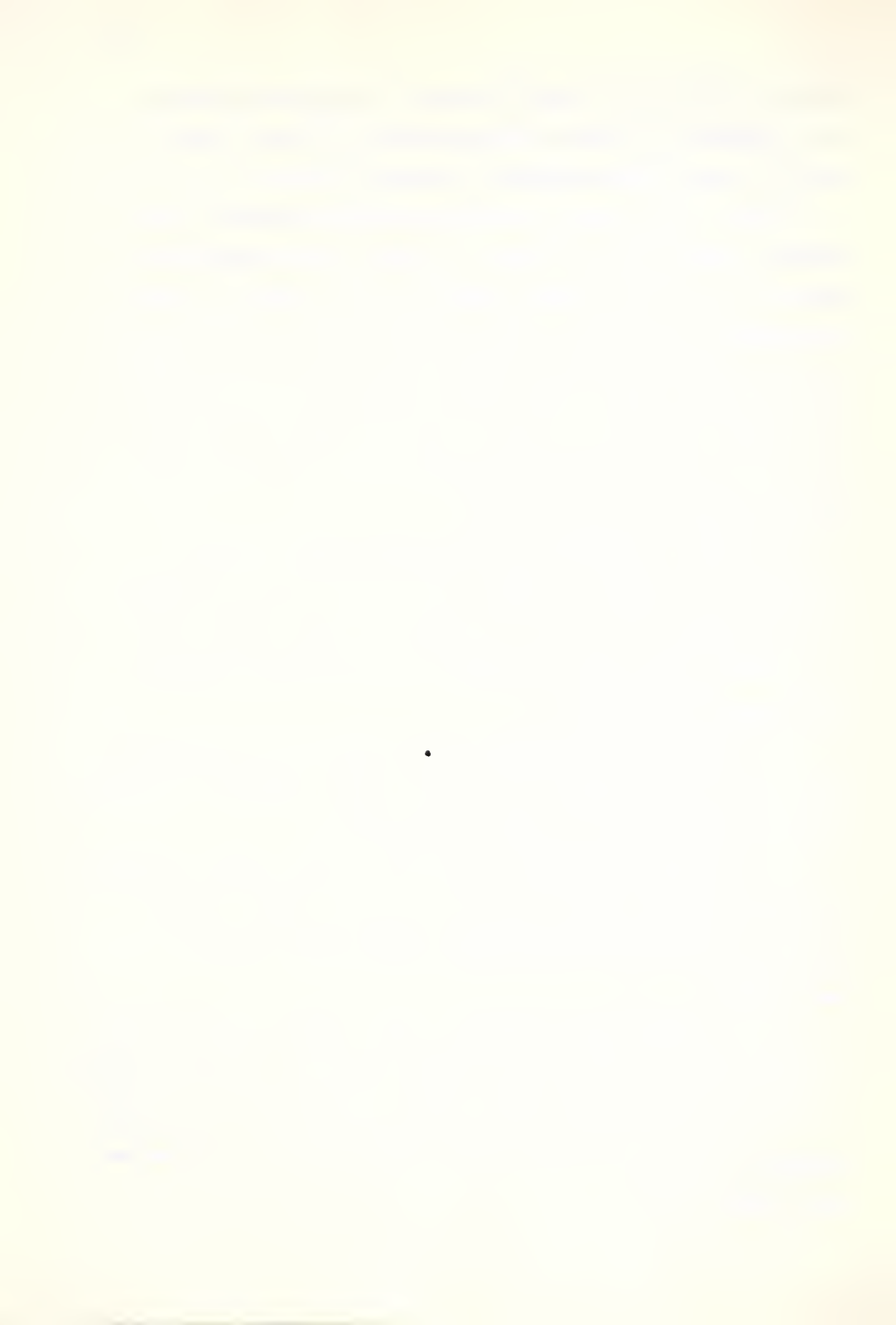


rugosity factor R. Also, shape did not have a noticeable influence. It is difficult to say much about the effects of  $D_s$  ratios larger than 3.4, since the rocks used did not exceed this value.

In this study the particle volume distributions used were rather narrow, or practically one-volume. It is apparent that mixed-volume particles will have more complex packing behavior. Differences will be introduced by the variations in rugosity R with particle volume and the relative amounts of each volume fraction. However, it is expected that the concepts of packing volume can be extended without much difficulty to the case of multi-size particles probably down to the filler size. This still remains to be demonstrated.

The immediate practical goals behind this research included consideration of aggregates "grading" close to those used in "one-size" and open-graded mixes, such as is sometimes the case in subbase, base and possibly binder courses for highways. The following translations into practice are pertinent:

- a) One-size crushed rock and one-size rounded gravel of identical sieve sizes are not identical when graded by volume. Response to compaction and service performance is expected to be different.
- b) If different aggregates were graded by particle volume, vibratory compaction in thick layers should require identical effort and their performance may not be greatly different provided other conditions were similar.
- c) If a binder, such as asphalt or clay, is added, rugosity as well as lubricating binder has to be considered in obtaining identical workability.
- d) The packing volume concept appears to offer promise as a unified mix design approach for diverse types of aggregates used in construction and research studies.



a) Grading by sieve size is expected to be useful for obtaining aggregate gradation by volume.

### CONCLUSIONS

These conclusions are based on laboratory measurements and analysis of certain crushed limestone, crushed gravel and rounded gravel aggregates of three particle volumes: 4 cc, 0.4 cc and 0.04 cc (about 3/4", 3/8" and 1/8" respectively). Although on this basis several important aggregate variables have been included on a fairly broad scale and it is probable that these conclusions can be applied to a wider range of aggregates than those studied, extension of the validity of the findings beyond the specific scope of this study remains to be demonstrated.

1. Particle packing volume, the volume which a piece of aggregate occupies in a mass of particles, is a parameter unifying the bulk behavior of the coarse aggregates tested.

2. The packing volume can be quantitatively defined by:

- (a) particle geometry and "surface" area, and
- (b) rugosity of the particle.

3. Rugosity includes primarily surface roughness, some accessible pores, plus some angularity of a particle. It is directly proportional to the volume of a given rock type.

4. Geometry of irregular particles can be satisfactorily characterized by an ellipsoid.

5. One volume ("size") ellipsoids ideally pack like spheres, giving porosities identical to those of spheres, regardless of size and dimensions.



6. The three rocks, with three sizes each, have identical porosities with identical total packing volumes  $\leq V_p$  when:

- a) poured into a container under identical conditions, or
- b) compacted by vibratory compaction under identical conditions.





#### ACKNOWLEDGMENTS

The authors wish to acknowledge the assistance of the Bureau of Public Roads, U.S. Department of Transportation, Federal Highway Administration and the Indiana State Highway Commission, who supported the investigation.

Laboratory testing was carried out in the Bituminous Materials Laboratory of the Joint Highway Research Project at Purdue University, Lafayette, Indiana.

# MEMORANDUM

The subject of this report is the results of the investigation conducted by the Department of the Interior, Bureau of Land Management, in connection with the proposed construction of a dam on the Colorado River at the mouth of the Grand Canyon. The purpose of the investigation was to determine the feasibility of the project and to estimate the cost of construction. The results of the investigation are as follows:

The proposed dam is located at the mouth of the Grand Canyon, where the Colorado River enters Lake Mead. The dam is to be constructed of concrete and will be 1,000 feet long. The height of the dam will be 200 feet. The dam will be constructed in two sections, each 500 feet long. The first section will be constructed first, and the second section will be constructed after the first section is completed. The dam will be constructed on a foundation of bedrock. The foundation will be excavated to a depth of 10 feet. The dam will be constructed on a foundation of bedrock. The foundation will be excavated to a depth of 10 feet. The dam will be constructed on a foundation of bedrock. The foundation will be excavated to a depth of 10 feet.

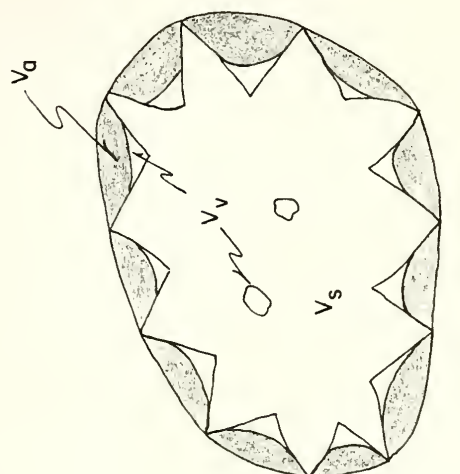
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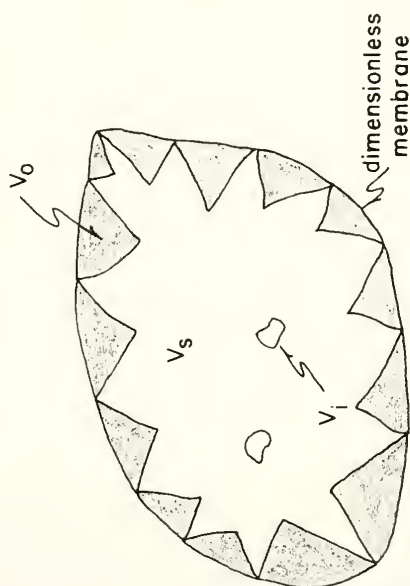
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Practical

$$V_p = V_s + V_v + V_d$$



Theoretical

$$V_p = V_s + V_i + V_o$$

Figure 1. Components of a particle packing volume





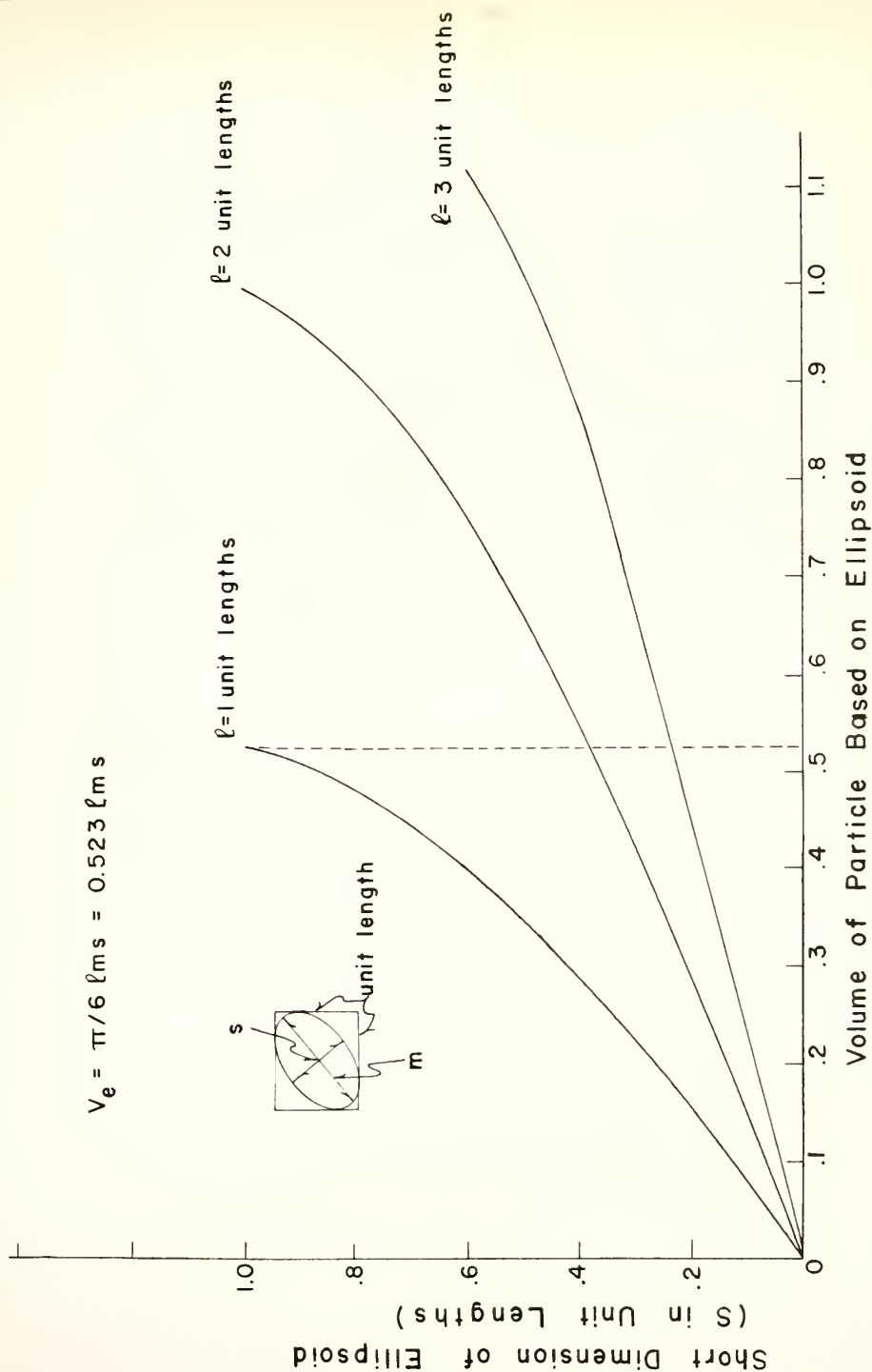
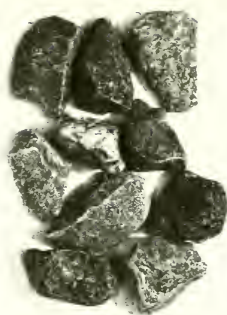


Figure 2. Theoretical volumes for ellipsoids passing a square sieve





Rounded Gravel, 4 cc  
RG-4



Crushed Gravel, 4 cc  
CG-4



Crushed Limestone, 4 cc  
CL-4



Rounded Gravel, 0.4 cc  
RG-0.4



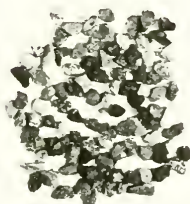
Crushed Gravel, 0.4 cc  
CG-0.4



Crushed Limestone, 0.4 cc  
CL-0.4



Rounded Gravel, 0.04 cc  
RG-0.04



Crushed Gravel, 0.04 cc  
CG-0.04



Crushed Limestone, 0.04 cc  
CL-0.04



Figure 3. Types and sizes of rocks used in this study



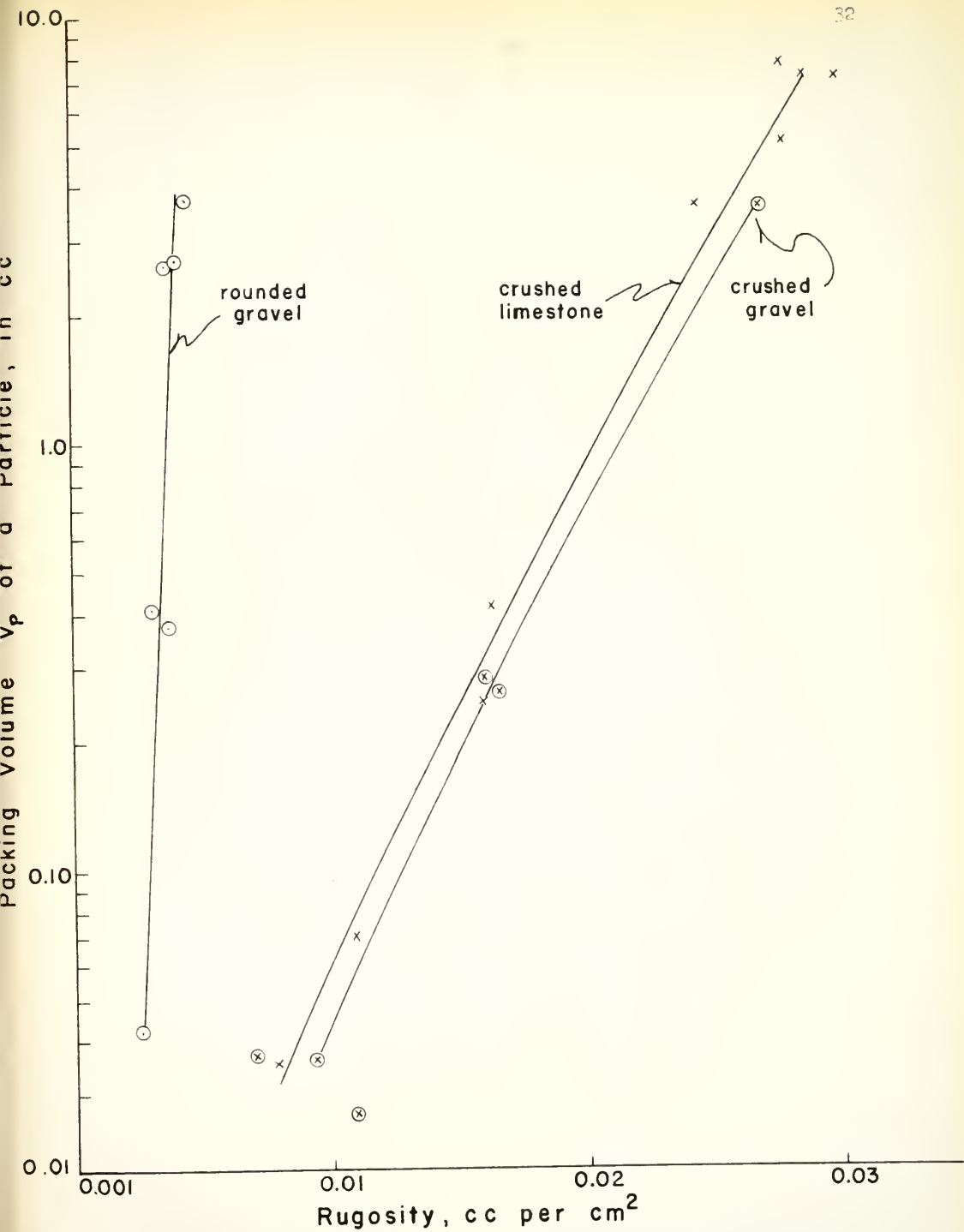


Figure 4. Rugosity versus packing volume for the three types of rocks



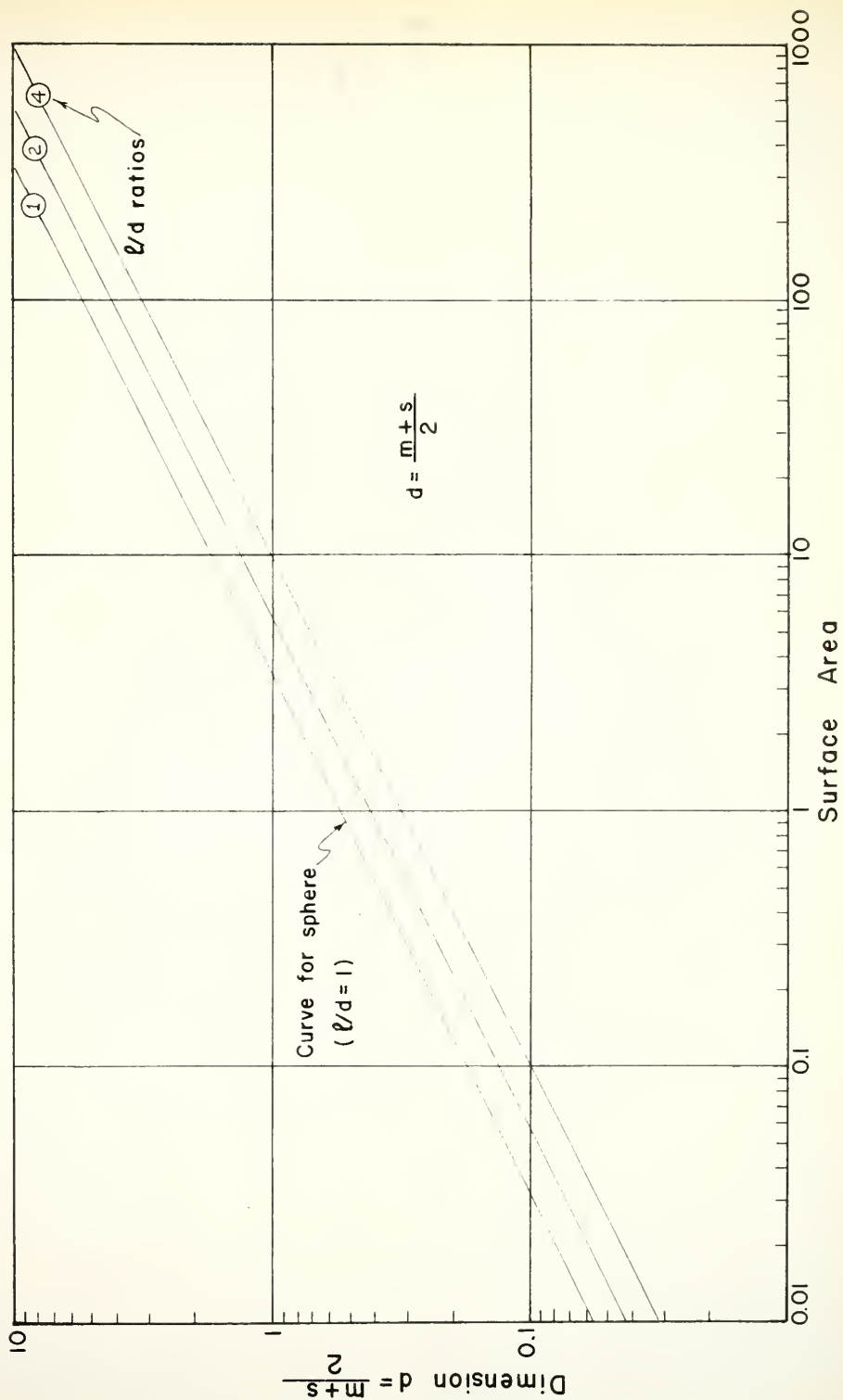


Figure 5. Graph for determining surface area for a prolate spheroid





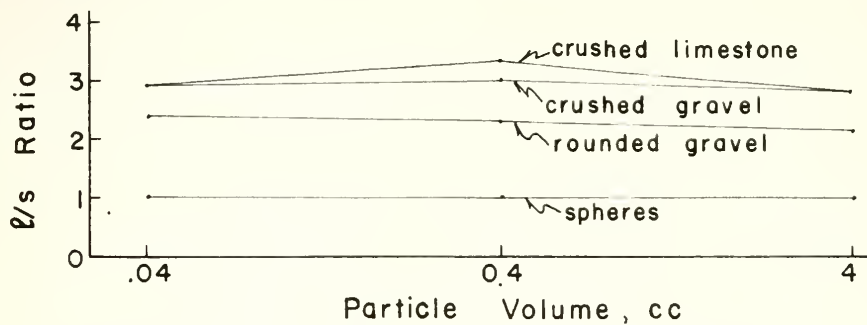


Figure 6a. Ratio  $l/s$  for various aggregates

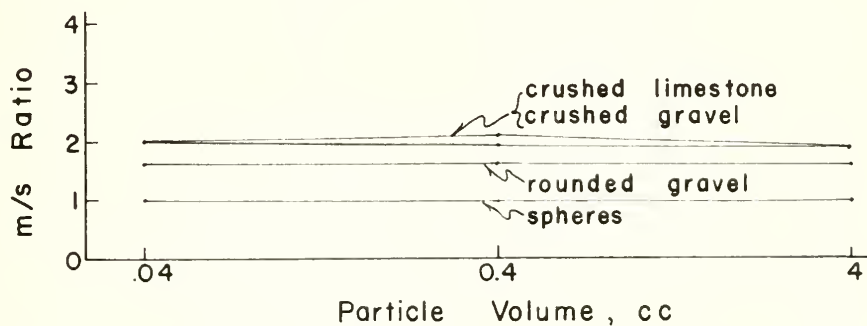


Figure 6b. Ratio  $m/s$  for various aggregates

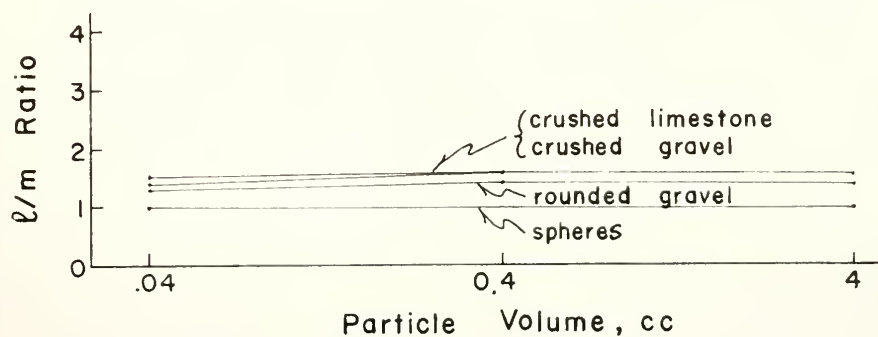


Figure 6c. Ratio  $l/m$  for various aggregates



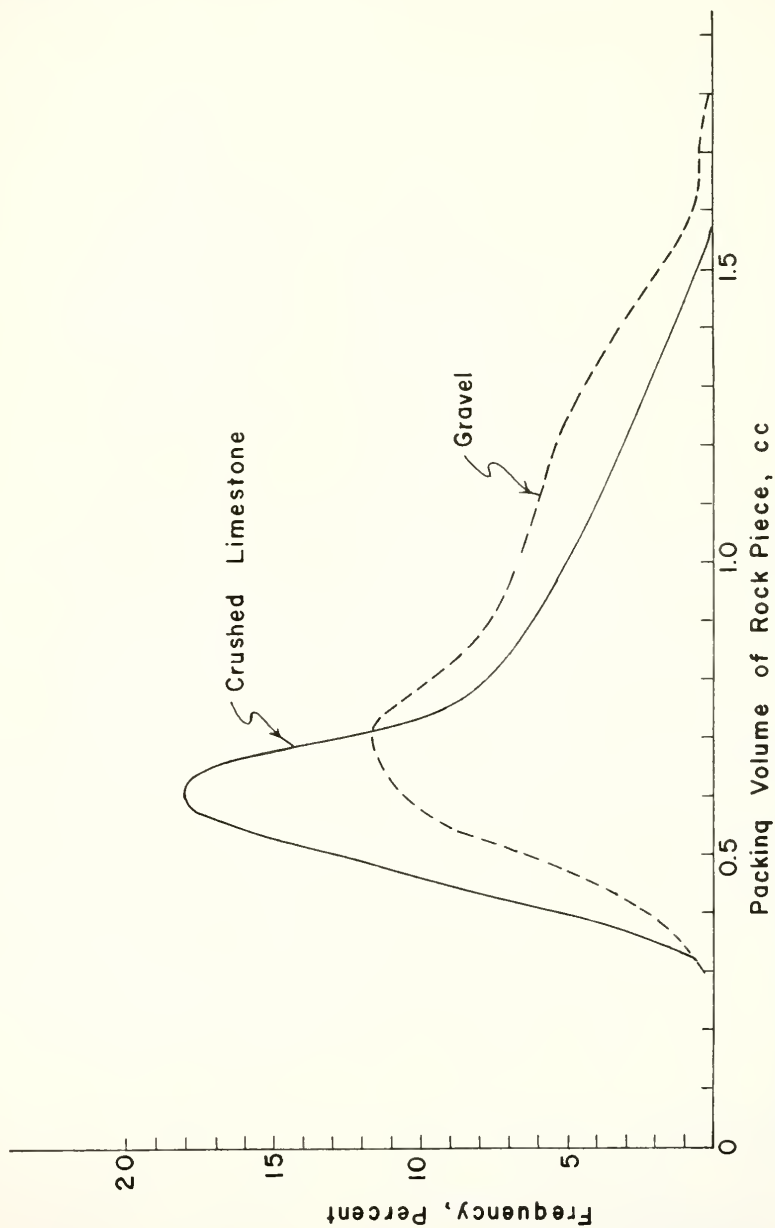


Figure 7. Example of particle volume distribution for 1/2"-3/8" crushed limestone and partially crushed (about 50% particles) gravel



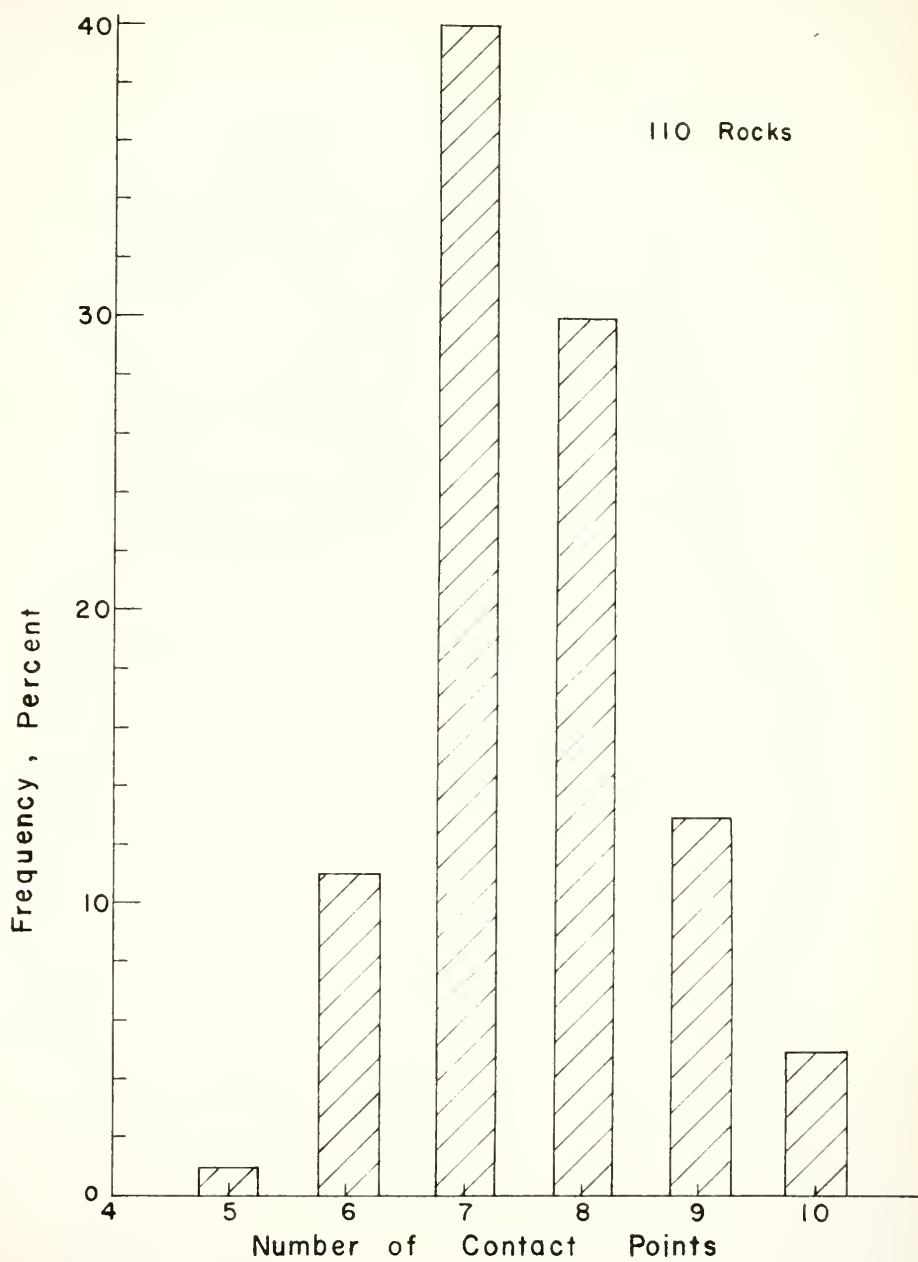


Figure 8. Example of contact point distribution, one-volume rocks



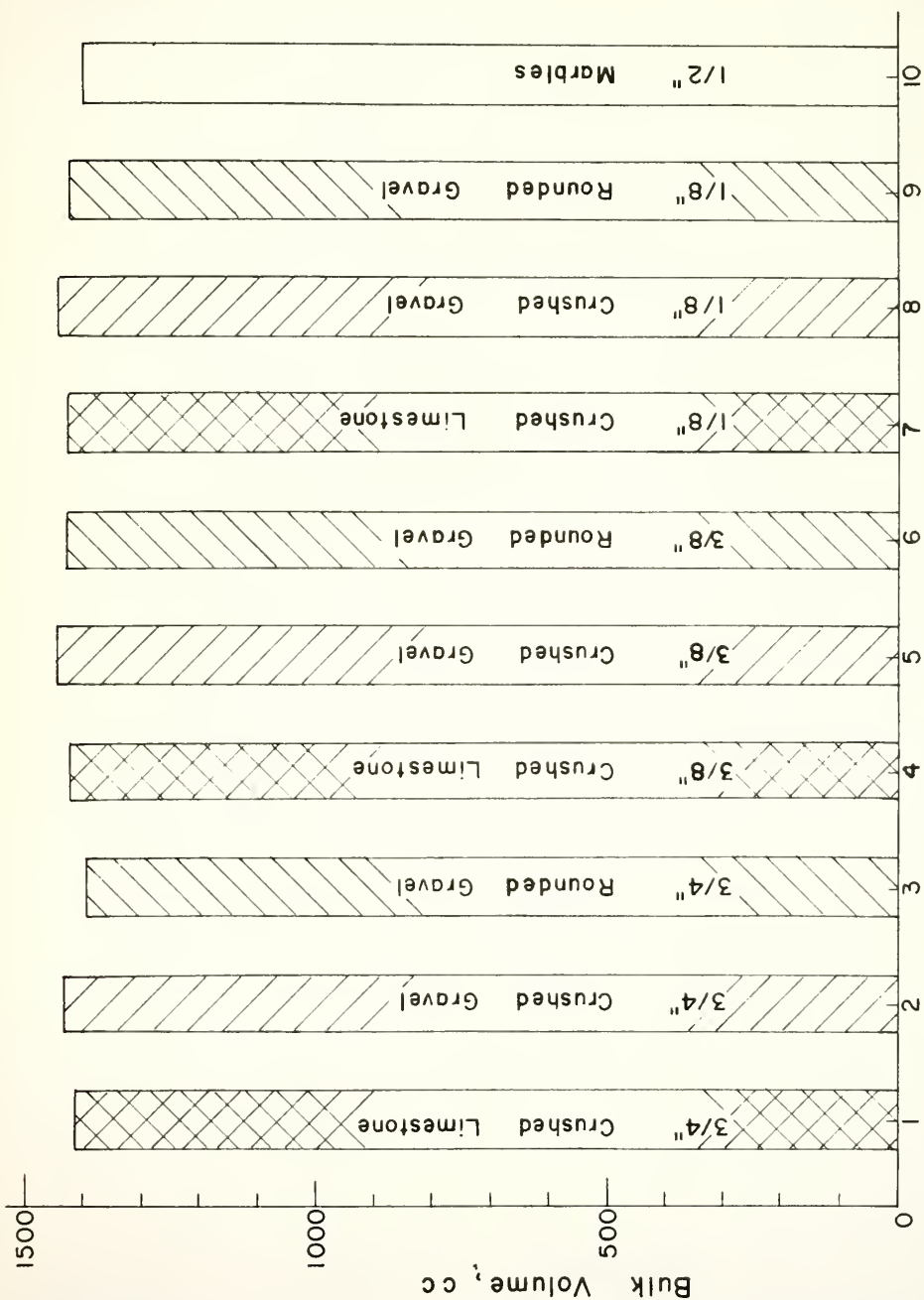


Figure 9 Loose volumes of  $\Sigma V_p = 800 \text{ cc}$ , 3" drop, 10 sec





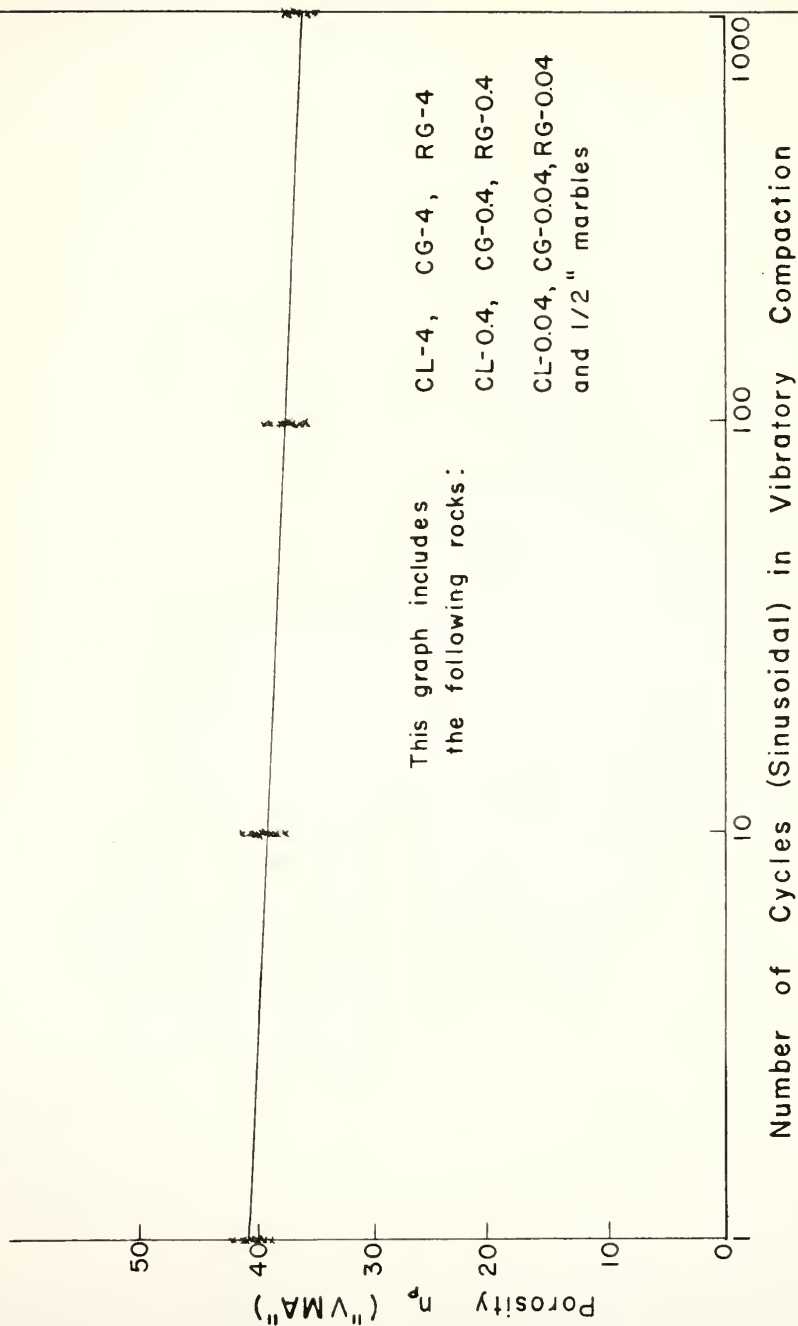
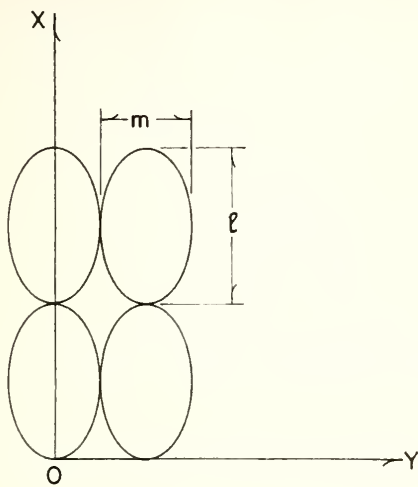
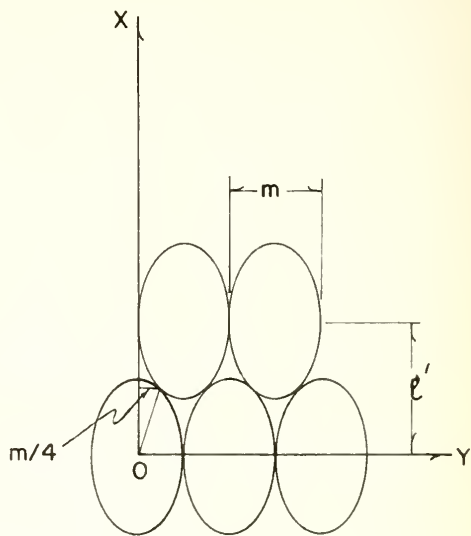


Figure 10. Porosity or voids in the bulk after various numbers of cycles at 1.5g's peak acceleration. All rocks and sizes plus 1/2" marbles are included.

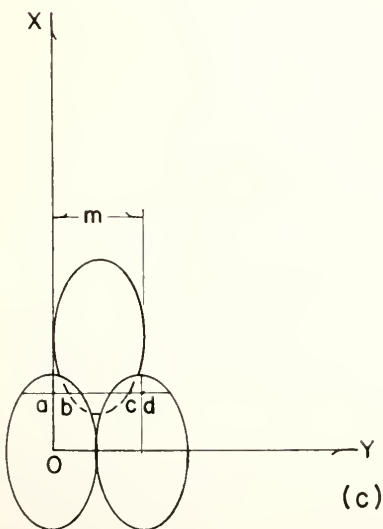




(a)



(b)



(c)

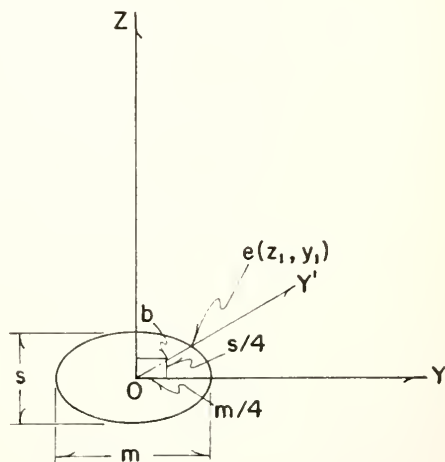
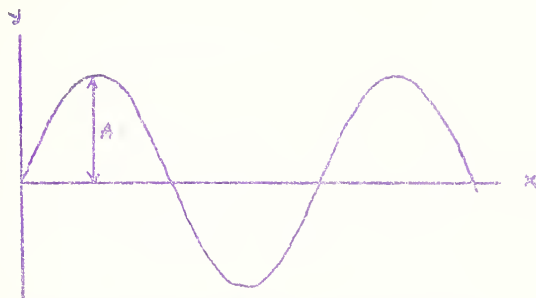


Figure 11. Packing of ellipsoids



## APPENDIX 1.

## VIBRATORY COMPACTION EQUATION



$x = A \sin wt$ , where  $w$  and  $t$  are angular velocity and time, respectively.

$$\frac{dy}{dx} = Aw \cos wt$$

$$\frac{d^2y}{dx^2} = Aw^2 \sin wt$$

for peak acceleration only:

$$\frac{d^2y}{dx^2} = -Aw^2$$

$$\text{acceleration in g's} = \frac{d^2y}{dx^2} / g = a_g$$

$$a_g = \frac{w^2 A}{g} = \frac{(2\pi f)^2 A}{g} = \frac{4\pi^2 f^2 A}{g}$$

$$f^2 = \frac{a_g}{\frac{4\pi^2}{g} A} = \frac{a_g}{.102A} \quad f^2 = \frac{a_g}{.102A}$$

$$\text{if } f = 20 \text{ cps, } a_g = 1.5 \text{ g's}$$

$$\text{amplitude should be } A = \frac{1.5}{(400)(0.103)} = .0364 \text{ inches}$$



## APPENDIX 2

### POROSITY OF PACKED ONE-SIZE ELLIPSOIDS

#### (a) Ellipsoids in "Cubical" Packing

As seen from Figure 11 part (a);

$$n = 1 - \frac{\pi}{6} \ell \sin \left( \frac{1}{\ell} \times \frac{1}{m} \times \frac{1}{s} \right) = .476 \text{ or } 47.6\%$$

#### (b) Ellipsoids in "Cubic-Tetrahedral" Packing

As seen from Figure 11 part (b);

$$n = 1 - \frac{\pi}{6} \ell \sin \left( \frac{1}{\ell} \times \frac{1}{m} \times \frac{1}{s} \right)$$

here  $\ell' < \ell$ ; now taking left corner ellipsoid

$$\frac{4x^2}{\ell'^2} + \frac{4y^2}{m^2} = 1 \quad y = \frac{\pi}{\ell'}$$

Substituting y

$$x = \frac{\sqrt{3}}{4} \ell'$$

$$\ell' = 2x = \frac{\sqrt{3}}{2} \ell$$

$$n = 1 - \frac{\pi}{6} \ell \sin \left( \frac{2}{\ell\sqrt{3}} \times \frac{1}{m} \times \frac{1}{s} \right) = .395 \text{ or } 39.5\%$$

#### (c) Ellipsoids in "Dense" Packing

As seen from Figure 11 part (c);

$$\frac{4x^2}{s^2} + \frac{4y^2}{m^2} = 1 \text{ for the left lower corner ellipsoid. Also}$$

$$\frac{z_1}{y_1} = \frac{s}{m}$$

$$z_1 = \frac{y_1 s}{m} \quad z_1^2 = \frac{y_1^2 s^2}{m^2}$$





Substituting  $z$  in the equation above

$$y_1 = \frac{\sqrt{2}}{4} m$$

$$z_1 = \frac{\sqrt{2}}{4} s$$

Now,

$$OE = R = \sqrt{y_1^2 + z_1^2} = \frac{\sqrt{2}}{4} \sqrt{m^2 + s^2}$$

$$OB = K_1 = \frac{1}{4} \sqrt{m^2 + s^2}$$

The ellipse going through  $OE \perp$  to  $yz$  plane can have an equation:

$$\frac{4x^2}{\ell^2} + \frac{y^2}{k^2} = 1$$

Since

$$y^1 = K_1$$

$$\frac{4x^2}{\ell^2} + \frac{(K_1)^2}{k^2} = 1$$

and

$$x = \frac{\sqrt{2}}{4} \ell$$

but

$$\ell' = 2x = \frac{\sqrt{2}}{2} \ell$$

and

$$n = 1 - \frac{\pi}{6} \ln \left( \frac{\sqrt{2}}{\ell} \times \frac{1}{m} \times \frac{1}{s} \right) = .260 \text{ or } 26.0\%$$



## APPENDIX 3

## COMPARISON OF MEASURED ELLIPSOID

## VOLUMES WITH VOLUMES BY WATER DISPLACEMENT METHOD

Numbers are percentages of difference.

Rock Size	→ CL	CG	EG	Tl.	n	$\sum x_{ij}^2$
	+ 10.8	+ 2.0	+ 5.3			
	- 16.8	- 6.1	- 8.3			
	+ 9.9	+ 1.3	+12.6			
	+ .3	-11.2	- 7.4			
4	- 8.3	- 4.4	+10.4	10.1	30	764.1
	- 8.7	+ 2.1	-11.7			722.0
	- 1.0	- 8.6	+ 9.4			<u>822.7</u>
	+ 2.8	+17.9	-12.7			2308.8
	+ 5.4	+11.6	- 0.8			
	+ 9.2	+ 1.0	+ 4.1			
	- 3.2	-12.1	+ 7.0			
	- 7.1	+ 8.9	- 5.8			
	+ 7.9	+10.5	-11.9			
	- 1.4	+ 6.2	- 0.3			
	+ 1.3	- 2.9	- 6.3			
0.4	- 11.8	- 2.6	+13.4	7.7	30	741.2
	- 6.8	+ 7.6	+ 3.8			575.9
	+ 1.4	- 3.6	+ 6.5			<u>640.2</u>
	+ 4.8	-10.7	-10.9			1957.3
	+ 20.1	+ 1.1	+ 4.6			



	+ 9.9	- 8.0	+ 5.5		
	- 18.3	+ 2.2	- 2.8		
	+ 7.6	+ 4.6	+16.2		
	- 12.1	- 7.2	+ 5.8		
	- 1.3	+11.4	- 6.2		
0.04	+ 3.1	+ 3.2	- 6.5	-2.4	30
	- 1.3	- 2.0	- 7.1		
	+ 2.6	+ 2.8	- 6.1		
	+ 0.6	- 1.3	- 7.4		
	+ 10.5	- 5.8	+ 9.0		
					767.6
					329.2
					<u>601.0</u>
					1697.8

$T_{.j}$	10.1	7.9	- 2.6	$T_{..} = 15.4$
$n$	30	30	30	$N = 90$
$\sum x_{.j}^2$	764.1	722.0	822.7	
	741.2	575.9	640.2	
	767.4	329.2	601.0	
	2272.7	1627.1	2063.9	$\sum \sum x_{.j}^2 = 5963.7$

Volume Difference Analysis

for C1-4 Stone

Model:  $d_i = \bar{D} + e$  (Correlated Samples)

H:  $\bar{D} = 0$

$$\begin{aligned} \bar{D} &= .36 \\ S_d^2 &= \frac{\sum d^2}{10} \\ S_d^2 &= \frac{764.1 - \frac{(3.6)^2}{10}}{9} = 84.8 \\ S_d &= \sqrt{\frac{84.8}{10}} = 2.9 \\ t &= \frac{\bar{d}}{S_d} = \frac{.36}{2.9} = .12 \end{aligned}$$

$t_9 (5) = 2.25 \quad .12 < 2.25 \quad \text{Hypothesis accepted}$



Similar results were obtained for:

	CG-4	RG-4
CL-0.4	CG-0.4	RG-0.4
CL-0.04	CG-0.04	RG-0.04

### Volume Difference Analysis

for CL-4, CL-0.4 and CL-0.04 Combined

Model:  $d_i = \bar{D} + \epsilon$

H:  $\bar{D} = 0$

$$\bar{D} = .34$$

$$s_d^2 = \frac{s^2}{30}$$

$$s^2 = \frac{2272.7 - \frac{(10.1)^2}{30}}{29} = \frac{2266.3}{29} = 78.2$$

$$s_d = \sqrt{\frac{78.2}{30}} = \sqrt{2.6} = 1.61$$

$$t = \frac{.34}{1.61} = .211$$

$$t_{29(5)} = 2.04$$

$$.211 < 2.04$$

Hypothesis accepted

Similar results were obtained for:

CG-4, CL-0.4, and CL-0.04 combined

and

RG-4, RG-0.4, and RG-0.04 combined





Volume Difference Analysis  
for CL-4, CG-4 and RG-4 Combined

Model:  $d_i = \bar{D} + \epsilon$

H:  $\bar{D} = 0$

$$\bar{D} = .34$$

$$s_d = \sqrt{\frac{s^2}{30}}$$

$$s^2 = \frac{2309 - \frac{(10.1)^2}{30}}{29} = \frac{2305.6}{29} = 80$$

$$s_d = \sqrt{\frac{80}{30}} = 1.63$$

$$t = \frac{.34}{1.63} = .21$$

$$t_{29(5)} = 2.04$$

$$.21 < 2.04$$

Hypothesis is accepted

Similar results were obtained for:

CL-0.4, CG-0.4 and RG-0.4 combined

and

CL-0.04, CG-0.04 and RG-0.04 combined

Revised Summary Report

for 1954-55 and 1955-56

Revised Summary Report

Volume Difference Analysis

## All Nine Combinations

$$\text{Model: } d_i = \bar{D} + \epsilon$$

$$H: \quad \bar{D} = 0$$

$$\bar{d} = \frac{.17}{s_d} = \sqrt{\frac{s^2}{90}}$$

$$s^2 = \frac{5964 - \frac{(15.4)^2}{90}}{89} = \frac{5561}{89} = 67$$

$$s_d = \sqrt{\frac{67}{90}} = \sqrt{.75} = .87$$

$$t = \frac{\bar{d}}{s_d} = \frac{.17}{.87} = .20$$

$$t_{89(5)} = 1.99$$

$$.20 < 1.99$$

Hypothesis accepted

Private Office and Assistant  
All these combinations

10-10-11

11-11-11

12-12-12

13-13-13

14-14-14

APPENDIX 4  
ANALYSIS OF  $\ell/s$  RATIOS

Hypothesis: Average  $\ell/s$  ratios for CL are equal:

	Size of Rock			
	<u>1</u>	<u>0.4</u>	<u>0.04</u>	
	3.3	1.9	2.5	
	2.0	4.3	3.2	
	2.1	4.3	2.5	
	2.7	3.5	3.4	
	3.1	3.7	2.5	
	2.5	3.7	3.5	
	2.6	2.4	4.4	
	2.0	3.7	3.6	
	2.9	3.4	2.1	
	<u>1.9</u>	<u>5.5</u>	<u>2.1</u>	
$T_{.j}$	25.1	36.4	29.8	$T_{..} = 91.3$
$n$	10	10	10	$N = 30$
$\sum x_{ij}^2$	65.2	141.5	93.9	$\sum \sum x_{ij}^2 = 300.6$

$$SS_{\text{total}} = \sum (x_{ij})^2 - \frac{T_{..}^2}{N} = 300.6 - \frac{(91.3)^2}{30} = 22.8$$

$$SS_{\text{treat.}} = \sum (T_{.j})^2 / n - \frac{(T_{..})^2}{N} = \frac{(25.1)^2 + (36.4)^2 + (29.8)^2}{10} - \frac{(91.3)^2}{30} = 6.5$$

$$SS_{\text{err.}} = SS_{\text{tot.}} - SS_{\text{treat.}} = 22.8 - 6.5 = 16.3$$



$$F_{2,27} = \frac{6.5}{16.3} = .4$$

$$F_{2,27}(.95) = 3.37$$

$$.4 < 3.37$$

Hypothesis accepted

Similar results were obtained for  $\ell/s$  ratios,  $\ell/m$  ratios and  $m/s$  ratios for all other rocks.





## APPENDIX 5

## ANALYSIS FOR NUMBER OF CONTACT POINTS

Hypothesis: Average number of contact points are equal in the following rocks:

CL-4	CL-0.04	RG-4	RG-0.04	Marbles	
(Number of Contact Points)					
7.6	7.4	7.6	7.0	7.6	
8.0	7.8	7.2	7.8	8.2	
7.2	8.6	7.4	7.6	7.2	
7.6	7.4	7.2	7.6	8.0	
Coded $X' = \frac{X-7.6}{.1}$			$\bar{X} = 7.6$		
0	-2	0	-6	0	
4	+2	+4	+2	+6	
-4	+10	-2	0	-4	
0	-2	-4	0	+4	
$T_j$	0	+8	-2	-4	+6
$n$	4	4	4	4	4
$\sum X_{ij}^2$	32	112	36	40	68
					$\sum \sum x_{ij}^2 = 288$

$$SS_{\text{total}} = 288 - \frac{64}{20} = 285$$

$$SS_{\text{treat}} = \frac{120}{4} - 3 = 27$$

$$SS_{\text{err}} = 261$$

$$F_{4,15} = .10$$

$$F_{4,15} (.95) = 3.06$$

$$.10 < 3.06$$

Hypothesis accepted



## APPENDIX 6

## ANALYSIS FOR POROSITIES IN VIBRATORY COMPACTION, 1 CYCLE

Hypothesis: The porosities  $n_p$  of all 10 batches are the same.

GL-L	GL-M	GL-S	CG-L	CG-M	CG-S	RG-L	RG-M	RG-S	MARB.
54	299	286	156	282	118	193	144	114	144
90	153	231	210	187	253	96	240	134	167
36	77	156	156	263	200	-4	164	173	167
198	131	156	156	319	146	36	202	54	121
127	153	213	56	148	227	76	164	94	121
505	813	1042	734	1199	944	397	914	569	720
5	5	5	5	5	5	5	5	5	5
67645	159309	229198	120244	307327	190778	53553	172932	72633	105796

$$T.. = 7837$$

$$N = 50$$

$$\text{Code: } X' = \frac{X - 39}{0.01}$$

$$L = 4$$

$$M = 0.4$$

$$S = 0.04$$

$$\sum \sum x_{ij}^2 = 1479415$$

$$SS_{\text{total}} = \sum (x_{ij})^2 - \frac{(T..)^2}{N} = 1479415 - \frac{(7837)^2}{50} = 251044$$

$$SS_{\text{treat}} = \sum \frac{(T..j)^2}{n} - \frac{(T..)^2}{N} = \frac{(505)^2 + (813)^2 + (1042)^2 + (734)^2 + (1199)^2 + (944)^2 + (397)^2 + (914)^2 + (569)^2 + (720)^2}{5} - \frac{(7837)^2}{50}$$

$$= \frac{(944)^2 + (397)^2 + (914)^2 + (569)^2 + (720)^2}{5} - \frac{(7837)^2}{50} = \frac{6704417}{5}$$

$$1228371 = 112512$$

$$SS_{\text{err}} = 251044 - 112512 = 138532$$

$$F_{9,40} = \frac{112512}{138532} = .8121 = \underline{\underline{.81}}$$

$$F_{9,40} (.95) = 2.12$$

$$.81 < 2.12$$

Hypothesis accepted



## ANALYSIS FOR POROSITIES IN VIBRATORY COMPACTION, 1000 CYCLES

Hypothesis: The porosities  $n_p$  of all 10 batches are the same.

GL-L	CL-M	CL-S	CG-L	CG-M	CG-S	RG-L	RG-M	RG-S	MARJ.
145	196	150	-58	229	106	97	300	224	155
42	23	85	28	106	290	120	126	91	232
21	111	85	71	195	260	97	192	114	331
63	89	172	50	239	74	120	170	158	181
145	154	-5	28	106	168	142	236	224	257
416	573	487	119	875	898	576	1024	811	1156
5	5	5	5	5	5	5	5	5	5
48224	82903	66559	12473	170059	196636	67782	227336	146593	286220

$$T.. = 6935$$

$$N = 50$$

$$\text{Code: } X' = \frac{\bar{X} - 3^c}{0.01}$$

$$\sum x_{ij}^2 = 1304785$$

$$SS_{\text{total}} = \sum (x_{ij})^2 - \frac{(T..)^2}{N} = 1304785 - \frac{(6935)^2}{50} = 342901$$

$$SS_{\text{treat}} = \sum \frac{(T..)^2}{n} - \frac{(T..)^2}{N} = \frac{(416)^2 + (573)^2 + (487)^2 + (119)^2 + (875)^2 + (898)^2 + (576)^2 + (1024)^2 + (811)^2 + (1156)^2}{5} - \frac{(6935)^2}{50}$$

$$= \frac{(1024)^2 + (811)^2 + (1156)^2}{50} - \frac{(6935)^2}{50} = 1139831 - 961884 = 177947$$

$$SS_{\text{err}} = 342901 - 177947 = 164954$$

$$F_{9,40} = \frac{177947}{164954} = 1.0787 = \underline{\underline{1.08}}$$

$$F_{9,40} (.95) = 2.12$$

$$1.08 < 2.12$$

Hypothesis accepted





